On the influence of the magnetic field of the GSI experimental storage ring on the time–modulation of the EC–decay rates of the H–like mother ions

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We investigate the influence of the magnetic field of the Experimental storage ring (ESR) at GSI on the periodic time–dependence of the orbital K–shell electron capture decay (EC) rates of the H–like heavy ions. We approximate the magnetic field of the ESR by a uniform magnetic field. Unlike the assertion by Lambiase *et al.*, arXiv: 0811.2302 [nucl–th], we show that a motion of the H–like heavy ion in a uniform magnetic field cannot be the origin of the periodic time–dependence of the EC–decay rates of the H–like heavy ions.

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INTRODUCTION

Recently Litvinov *et al.* [1] have observed that the K–shell electron capture (EC) decay rates of H–like $^{140}\mathrm{Pr}^{58+}$ and $^{142}\mathrm{Pm}^{60+}$ ions

$$\begin{aligned} &^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{58+} + \nu_e, \\ &^{142}\mathrm{Pm}^{60+} \to {}^{142}\mathrm{Nd}^{60+} + \nu_e \end{aligned} \qquad (1)$$

have an unexpected periodic time modulation of exponential decay curves. The rates of the number N^{EC}_d of daughter ions $^{140}\mathrm{Ce}^{58+}$ and $^{142}\mathrm{Nd}^{60+}$

$$\frac{dN^{EC}_d(t)}{dt} = \lambda_{EC}(t)\, N_m(t), \qquad (2)$$

where $N_m(t)$ is the number of the H–like mother ions $^{140}\mathrm{Pr}^{58+}$ or $^{142}\mathrm{Pm}^{60+}$ [1] and $\lambda^{(\mathrm{H})}_{EC}(t)$ is the EC–decay rate, are periodic functions, caused by a periodic time–dependence of the EC–decay rates

$$\lambda_{EC}(t) = \lambda_{EC}\,(1 + a^{EC}_d\,\cos(\omega_{EC}t + \phi_{EC})) \qquad (3)$$

with periods $T_{EC} = 2\pi/\omega_{EC} = 7.06(8)\,\mathrm{s}$ and $T_{EC} = 2\pi/\omega_{EC} = 7.11(22)\,\mathrm{s}$ for the EC–decays of $^{140}\mathrm{Pr}^{58+}$ or $^{142}\mathrm{Pm}^{60+}$, respectively, amplitudes $a^{EC}_d \simeq 0.20$ and phases ϕ_{EC}. Below such a periodic time–dependence we call the "GSI oscillations".

Recently [2]–[4] the decay rate of the EC–decay $^{122}\mathrm{I}^{52+} \to {}^{122}\mathrm{Te}^{52+} + \nu_e$ with a period of the time–modulation $T_{EC} = 6.11(3)\,\mathrm{s}$ has been observed. As has been pointed out in [2]–[4], the periods of the time–modulation of the H–like heavy ions obey the A-scaling: $T_{EC} = A/20\,\mathrm{s}$, where A is a mass number of the mother H–like heavy ions.

In the articles [5] (see also [6, 7]) we have proposed an explanation of the periodic time–dependence of the EC–decay rates as an interference of two neutrino mass–eigenstates ν_1 and ν_2 with masses m_1 and m_2, respectively. The period T_{EC} of the time–dependence has been related to the difference $\Delta m^2_{21} = m^2_2 - m^2_1$ of the squared neutrino masses m_2 and m_1 as follows

$$\omega_{EC} = \frac{2\pi}{T_{EC}} = \frac{\Delta m^2_{21}}{2\gamma M_m}, \qquad (4)$$

where γM_m is the energy of the H–like mother ion with mass M_m in the ESR and $\gamma = 1.43$ is a Lorentz factor [1]. In a subsequent analysis we also showed that the β^+–branches of the decaying H–like heavy ions do not show time modulation, because of the broad continuous energy spectrum of the neutrinos [5]. This agrees well with the experimental data [2]–[4].

According to the atomic quantum beat experiments and theory [8, 9], the interpretation of the "GSI oscillations", proposed in [5]–[7], bears similarity with quantum beats of atomic transitions, when an excited atomic eigenstate decays into a coherent state of two (or several) lower lying atomic eigenstates. In the case of the EC–decay one deals with a transition from the initial state $|m\rangle$ to the final state $|d\,\nu_e\rangle$, where the electron neutrino is a coherent superposition of two neutrino mass–eigenstates with the energy difference equal to $\omega_{21} = \Delta m^2_{21}/2M_m$ related to ω_{EC} as $\omega_{EC} = \omega_{21}/\gamma$.

As has been pointed out in [10], a motion of the H–like mother ion in the magnetic field of the ESR can be the origin of the "GSI oscillations" [1]. In this letter we investigate the influence of the magnetic field of the ESR at GSI making a consistent calculation of the EC–decay rate by using the weak interaction Hamilton operator and taking into account a motion of the mother H–like ion in the magnetic field. For simplicity we approximate the

magnetic field of the ESR at GSI by a constant magnetic field $\vec{B} = B_0\,\vec{e}_z$ directed perpendicular the plane of the ESR. However, we neglect also a possible quantisation of the energy of the mother H–like in the constant magnetic field [11] and take into account only the interaction of a spin of the mother H–like ion with a constant magnetic field. We show that such a spin–rotation coupling of the H–like heavy ions cannot be responsible for the periodic time–dependence of the EC–decay rates, measured at GSI [1]–[4].

The Hamilton of the interaction of the H–like ions with a magnetic field $\vec{B} = B_0\,\vec{e}_z$ we define as [12]

$$\begin{aligned} H_{\vec{B}} &= 2\Big(a_e + \frac{1}{\gamma}\Big)\,\mu_B \vec{s}\cdot\vec{B} \\ &- \Big(g_I - \frac{2Zm_p}{M_I}\Big(1 - \frac{1}{\gamma}\Big)\Big)\,\mu_N\,\vec{I}\cdot\vec{B}, \qquad (5) \end{aligned}$$

where $\vec{s} = \frac{1}{2}\,\vec{\sigma}$ and $\vec{I}$ are operators of spins of the electron and the mother nucleus with eigenvalues $s = \frac{1}{2}$ and $I = 1$; $a_e = (g_e - 2)/2$ is the anomalous magnetic moment of the bound electron with g_e equal to [13]–[15]

$$\begin{aligned} \frac{1}{2}\,g_e &= 1 + \frac{2}{3}\,(\sqrt{1 - (\alpha Z)^2} - 1) \\ &+ \frac{\alpha}{\pi}\Big(\frac{1}{2} + \frac{1}{12}\,(\alpha Z)^2 + \frac{7}{2}\,(\alpha Z)^4 + \ldots\Big), \qquad (6) \end{aligned}$$

where $Z = 59$ for the H–like heavy ion ${}^{140}\mathrm{Pr}^{58+}$; $g_I = \mu_I/I$ and M_I are the anomalous magnetic moment of the nucleus with spin I and the mass. For the nucleus ${}^{140}\mathrm{Pr}^{59+}$ they are equal to $g_I = 2.5$ [16] and $M_I = 130324.46\,\mathrm{MeV}$. Then, $\mu_B = e/2m_e = 5.788\times 10^{-5}\,\mathrm{eV\,T^{-1}}$ and $\mu_N = e/2m_p = 3.152\times 10^{-8}\,\mathrm{eV\,T^{-1}}$ are the Bohr and nuclear magnetons [17]; $\gamma = 1.43$ is a Lorentz factor of a motion of a H–like heavy ion in the ESR at GSI [1], the value of the magnetic field is $B_0 = 1.19703\,\mathrm{T}$ [1]. The terms, proportional to $(1 - 1/\gamma)$, come from the Thomas precession [12]. The electric charges of the interacting particles are defined in terms of the electric charge of the proton e.

For the calculation of the amplitude of the EC–decays $m \to d + \nu_e$ of the mother H–like heavy ion m we have to use a standard weak interaction Hamilton operator

$$\begin{aligned} &H_W(t) = \frac{G_F}{\sqrt{2}}V_{ud}\int d^3x[\bar{\psi}_n(x)\gamma^{\mu}(1 - g_A\gamma^5)\psi_p(x)] \\ &\times[\bar{\psi}_{\nu_e}(x)\gamma_{\mu}(1 - \gamma^5)\psi_e(x)] \qquad (7) \end{aligned}$$

with standard notations [18]. As has been shown in [6], the non–trivial contribution to the EC–decay rate of the H–like heavy ion in the ground $(1s)_{F=\frac{1}{2},M_F\pm\frac{1}{2}}$ comes from the state with the wave function $|t,(1s)_{\frac{1}{2},-\frac{1}{2}}\rangle$. This means that the evolution of the H–like heavy ion into the state $d + \nu_e$ is defined by the wave function $|t,(1s)_{\frac{1}{2},-\frac{1}{2}}\rangle$ only.

In the laboratory frame the evolution of the mother H–like ion m in time is described by the wave function $|t,(1s)_{\frac{1}{2},-\frac{1}{2}}\rangle$

$$\begin{aligned} |t,(1s)_{\frac{1}{2},-\frac{1}{2}}\rangle &= -e^{-iE^{(-+)}_m t}\sqrt{\frac{2}{3}}\,|1,-1\rangle|\frac{1}{2},+\frac{1}{2}\rangle \\ &+ e^{-iE^{(0-)}_m t}\sqrt{\frac{1}{3}}\,|1,0\rangle|\frac{1}{2},-\frac{1}{2}\rangle, \qquad (8) \end{aligned}$$

where $|I,I_z\rangle$ and $|s,s_z\rangle$ are spinorial wave functions of the nucleus and the electron of the H–like heavy ion with eigenvalues $I = 1, I_z = 0,\pm 1$ and $s = \frac{1}{2}, s_z = \pm\frac{1}{2}$, respectively. The energies $E^{(-+)}_m$ and $E^{(0-)}_m$ are defined by

$$\begin{aligned} E^{(-+)}_m &= E_m + \Big(a_e + \frac{1}{\gamma}\Big)\,\mu_B\,B_0\,\cos\theta_e \\ &+ \Big(g_I - \frac{2Zm_p}{M_I}\Big(1 - \frac{1}{\gamma}\Big)\Big)\,\mu_N\,B_0\,\cos\theta_I, \\ E^{(0-)}_m &= E_m - \Big(a_e + \frac{1}{\gamma}\Big)\,\mu_B\,B_0\,\cos\theta_e, \qquad (9) \end{aligned}$$

where $E_m = \gamma M_m - i\,\frac{1}{2}\,\lambda_m$ and λ_m is the weak decay rate of the H–like mother ion in the laboratory frame [17], θ_e and θ_I are the angles between the z–axis and the axes of quantisation of the spins of the electron and the nucleus, respectively. Since in the GSI experiments the H–like heavy ions are in the $(1s)_{F=\frac{1}{2}}$, the spins of the electron and the nucleus should be anti–parallel. This implies that $\cos\theta_e = -\cos\theta_I$.

Using the wave function $|t,(1s)_{\frac{1}{2},-\frac{1}{2}}\rangle$ the probability of finding a mother H–like heavy ion at time t is

$$\begin{aligned} &P_m(t;\theta_e) = e^{-\lambda_m t}\,|\langle(1s)_{\frac{1}{2},-\frac{1}{2}},t|0,(1s)_{\frac{1}{2},-\frac{1}{2}}\rangle|^2 = \\ &= e^{-\lambda_m t}\,\frac{5}{9}\,\Big(1 + \frac{4}{5}\cos(\omega_B\cos\theta_e t)\Big), \qquad (10) \end{aligned}$$

where the frequency ω_B is equal to

$$\begin{aligned} &\omega_B = \Big[\Big(2a_e + \frac{2}{\gamma}\Big) - \Big(g_I - \frac{2Zm_p}{M_I}\Big(1 - \frac{1}{\gamma}\Big)\Big)\frac{m_e}{m_p}\Big] \\ &\times\mu_B B_0 = 1.34\times 10^{11}\,\mathrm{s}^{-1}. \qquad (11) \end{aligned}$$

Due to the factor m_e/m_p the dominant contribution comes from the electron anomalous magnetic moment. A period of the time modulation is equal to

$$T_B = \frac{2\pi}{\omega_B} = 4.70\times 10^{-11}\,\mathrm{s}. \qquad (12)$$

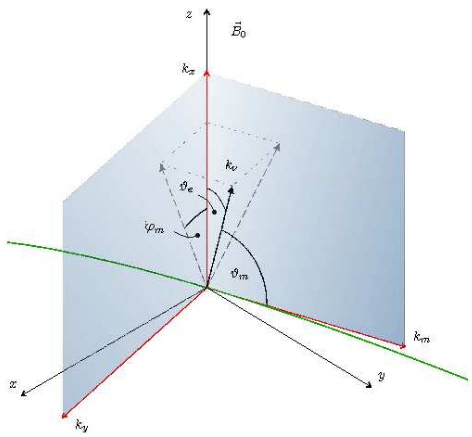

FIG. 1: Kinematical relations between the angles of the axis of quantisation of spins and momenta of interacting particles.

Practically, the period T_B is proportional to the electron mass. This disagrees with the experimental A–scaling of the period of the time modulation of the H–like ions, measured at GSI [2]–[4]. Such a period of the time modulation cannot be measured at the present level of the experimental time resolution at GSI [5]. The probability $P_m(t;\theta_e)$ should be averaged over the neutrino angular distribution, calculated in the laboratory frame and given by

$$\frac{dW_{\nu_e}}{d\Omega_m} = \frac{1}{8\pi\gamma^4}\,\frac{1}{(1 - v_m\cos\theta_m)^3}, \tag{13}$$

where $d\Omega_m = \sin\theta_m d\theta_m d\varphi_m$ is an element of the solid angle in the momentum space with axial axis directed along the momentum $\vec{k}_m$ of the mother ion. This gives

$$\begin{aligned} P_m(t) &= \int P_m(t;\theta_e)\,\frac{dW_{\nu_e}}{d\Omega_m}\,d\Omega_m = \\ &= e^{-\lambda_m t}\,\frac{5}{9}\,\frac{1}{8\pi\gamma^4}\int_0^{2\pi}\!\!\int_0^{\pi}\frac{\sin\theta_m d\theta_m d\varphi_m}{(1 - v_m\cos\theta_m)^3} \\ &\times \Big(1 + \frac{4}{5}\cos(\omega_B\sin\theta_m\cos\varphi_m t)\Big). \end{aligned} \tag{14}$$

As it is shown in Fig. 1, the angle θ_e is related to angles θ_m and φ_m as follows $\cos\theta_e = \sin\theta_m\cos\varphi_m$. Integrating over the azimuthal angle φ_m and using the integral representation Bessel functions and the properties of infinite series of Bessel functions [19] we get

$$\begin{aligned} P_m(t) &= e^{-\lambda_m t}\Big(1 - \frac{4}{9}\frac{1}{\gamma^4}\int_0^{\pi}\frac{d\theta_m\sin\theta_m}{(1 - v_m\cos\theta_m)^3} \\ &\times \sum_{n=1}^{\infty} J_{2n}(\omega_B\sin\theta_m t)\Big), \end{aligned} \tag{15}$$

This shows that the interaction of the mother H–like heavy ion with the uniform magnetic field of the storage ring cannot provide a time–modulation of the EC–decay rate, observed at the experiment [1].

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[1] Yu. A. Litvinov *et al.* (the GSI Collaboration), Phys. Lett. B **664**, (2008) 162.
[2] P. Kienle, A talk at International Conference on Particles And Nuclei (PANIC08), 9 - 14 of November, 2008 in Eilat, Israel (to appear in Nucl. Phys. A).
[3] P. Kienle, *Two–Body Weak Decay Studies in an Ion Storage Ring*, A talk at DISCRETE'08: Symposium on properties in physics of discrete symmetries, 11–16 December 2008, IFIC, Valencia, Spain URL: http://ific.uv.es/discrete08/, (to be published in J. Phys.: Conf. Ser. in third quarter of 2009).
[4] N. Winckler *et al.*, GSI Annual report 2008.
[5] A. N. Ivanov *et al.*, Phys. Rev. Lett. **101**, (2008) 182501.
[6] A. N. Ivanov *et al.*, arXiv: 0801.2121 [nucl–th].
[7] A. N. Ivanov *et al.*, arXiv: 0804.1311 [nucl–th].
[8] H. J. Andreä, Physics Scripta **9**, (1974) 257; Phys. Rev. A **2**, (1970) 2200.
[9] W. W. Chow *at al.*, Phys. Rev. A **11**, (1975) 1380.
[10] G. Lambiase, G. Papini, and G. Scarpetta, arXiv: 0811.2302 [nucl–th].
[11] L. D. Landau and E. M. Lifschitz, in *Lehrbuch der Theoretischen Physik*, Band III *Quantenmechanik*, Verlag Harri Deutsch, pp.442, 2007.
[12] J. D. Jackson, Rev. Mod. Phys. **48**, 417 (1976); J. D. Jackson, in *Klassische Elekrodynamik*, Walter de Gruyter, Berlin, 1983.
[13] H. A. Bethe and E. E. Salpeter, in *Quantum mechanics of one– and two–electron atoms*, Springer–Verlag, Berlin, p. 213, 1957.
[14] S. A. Blundell, K. T. Cheng, and J. Sapirstein, Phys. Rev. A **55**, (1997) 1857.
[15] D. L. Moskovkin *et al.*, Phys. Rev. A **70**, (2004) 032105.
[16] Yu. A. Litvinov *et al.* (the GSI Collaboration), Phys. Rev. Lett. **99**, (2008) 262501.
[17] C. Amsler *et al.*, Phys. Lett. B **667**, (2008) 1.
[18] A. N. Ivanov *et al.*, Phys. Rev. C **78**, (2008) 025503; arXiv: 0711.3184 [nucl–th].
[19] *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, Ed. by M. Abramowitz and I. A. Stegun, National Bureau of Standards, Applied Mathematics Series • 55, 1972.